Example Candidate Responses

Cambridge International AS & A Level

Cambridge International AS and A Level Mathematics

9709

Paper 6

Cambridge Advanced

CAMBRIDGE International Examinations

Version 1.0

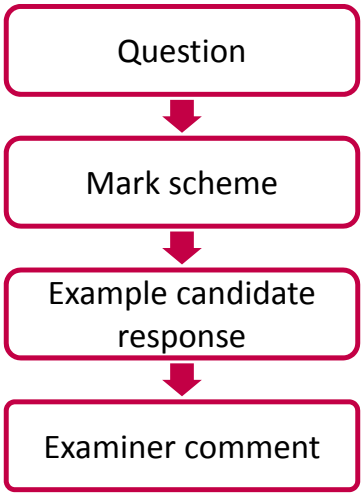
Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics (9709), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



Each question is followed by an extract of the mark scheme used by examiners. This, in turn, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at https://teachers.cie.org.uk

Assessment at a glance

The 7 units in the scheme cover the following subject areas:

- Pure Mathematics (units P1, P2 and P3);
- Mechanics (units M1 and M2);
- Probability and Statistics (units S1 and S2).

Centres and candidates may:

- take all four Advanced (A) Level components in the same examination session for the full A Level.
- follow a staged assessment route to the A Level by taking two Advanced Subsidiary (AS) papers (P1 & M1 **or** P1 & S1) in an earlier examination session;
- take the Advanced Subsidiary (AS) qualification only.

AS Level candidates take:

Paper 1: Pure Mathematics 1 (P1)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 60% of total

plus **one** of the following papers:

Paper 2: Pure Mathematics 2 (P2)	Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 40% of total

A Level candidates take:

Paper 1: Pure Mathematics 1 (P1)	Paper 3 Pure Mathematics 3 (P3)
1¾ hours About 10 shorter and longer questions 75 marks weighted at 30% of total	**1¾ hours** About 10 shorter and longer questions 75 marks weighted at 30% of total

plus **one** of the following combinations of two papers:

Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics 1 (S1)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 4: Mechanics 1 (M1)	Paper 5: Mechanics 2 (M2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

or

Paper 6: Probability and Statistics 1 (S1)	Paper 7: Probability and Statistics 2 (S2)
1¼ hours About 7 shorter and longer questions 50 marks weighted at 20% of total	**1¼ hours** About 7 shorter and longer questions 50 marks weighted at 20% of total

Teachers are reminded that the latest syllabus is available on our public website at **www.cie.org.uk** and Teacher Support at **https://teachers.cie.org.uk**

Paper 6

Question 1

1 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

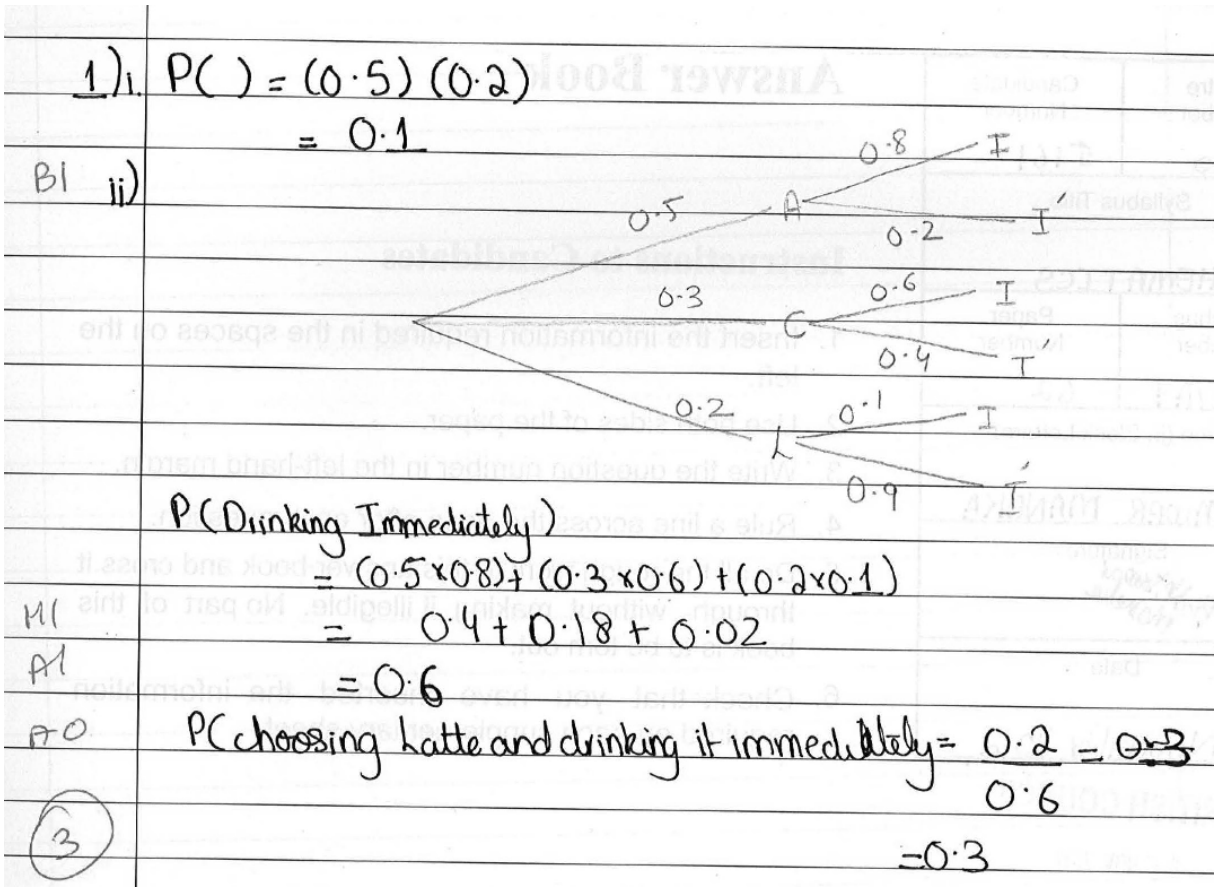
(i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

(ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. [4]

Mark scheme

1	**(i)** P (A Later) = 0.5 × 0.2 = 0.1	B1	[1]	
	(ii) P(L given I) = (0.2 × 0.1)/(0.5 × 0.8 + 0.3 × 0.6 + 0.2 × 0.1)	B1		0.2 × 0.1 seen on its own as num or denom of a fraction
		M1		Attempt at P(I) summing 2 or 3 2-factor prods, seen anywhere
	= 0.02/0.6	A1		Correct unsimplified P(I) as num or denom of a fraction
	= 0.0333 (1/30)	A1	[4]	Correct answer accept 0.033

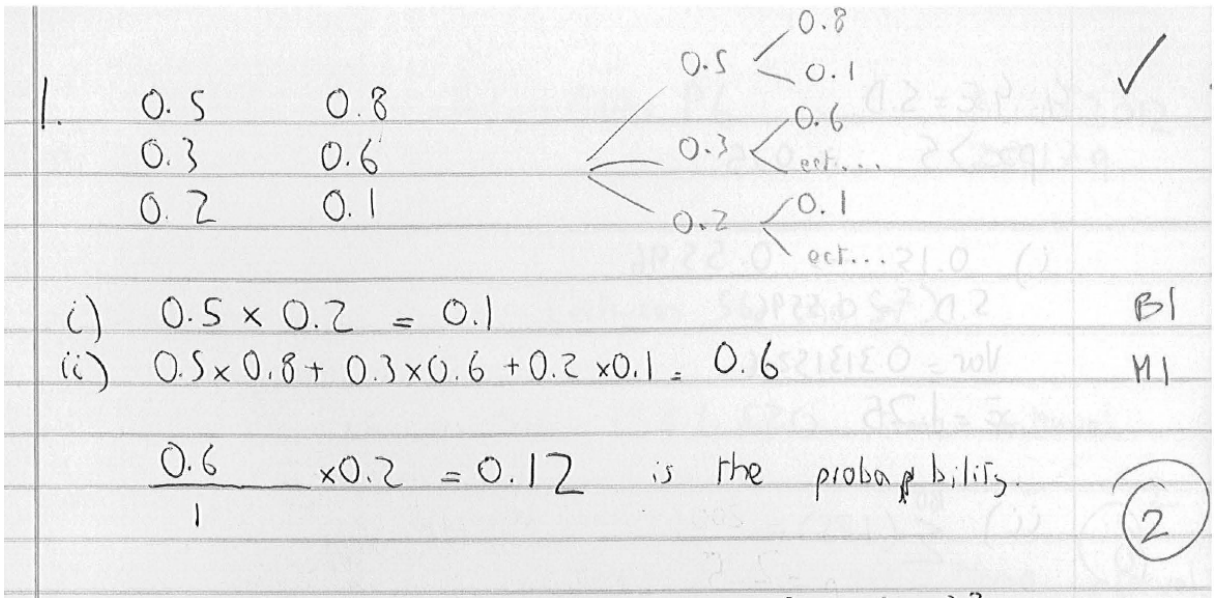
Example candidate response – 1



Item marks awarded: (i) = 1/1; (ii) = 2/4

Total mark awarded = 3 out of 5

Example candidate response – 2



Item marks awarded: (i) = 1/1; (ii) = 1/4

Total mark awarded = 2 out of 5

Examiner comment – 1 and 2

(i) This was a routine first part of the first question and both candidates answered it correctly.

(ii) Both candidates recognised that the probability P (drinking immediately) was required and both found this probability correctly. Candidate 1 recognised that this was part of a conditional probability question and used this value as the denominator of the associated conditional probability fraction. However, the numerator was not correct. Candidate 2 was unable to proceed further after having found P (drinking immediately).

Question 2

2 The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

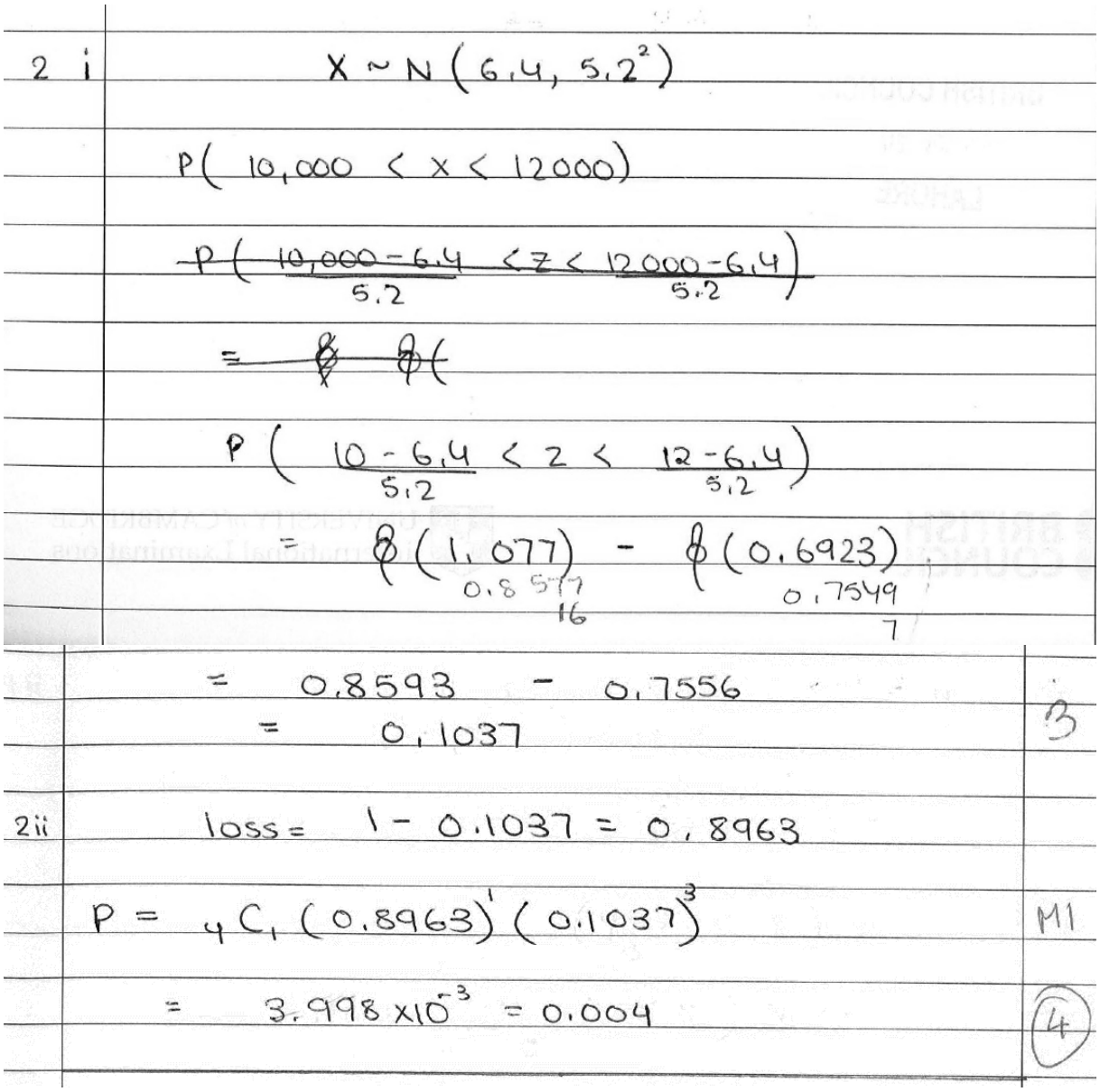
(i) Find the probability that, on a randomly chosen day, the company makes a profit between \$10 000 and \$12 000. [3]

(ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days. [4]

Mark scheme

2	**(i)** $z_1 = \frac{12-6.4}{5.2} = 1.077$	M1		Standardising, can be all in thousands, no mix, no cc no sq rt no sq
	$z_2 = \frac{10-6.4}{5.2} = 0.692$	M1		$\Phi_2 - \Phi_1$, Φ_2 must be $> \Phi_1$
	$\Phi(z_1) - \Phi(z_2) = 0.8593 - 0.7556$ $= 0.104$	A1	[3]	Correct answer
	(ii) P(loss) $= P(z < \frac{0-6.4}{5.2}) = P(z < -1.231)$ $= 1 - 0.8909$	M1		Standardising using $x = 0$, accept $\frac{0.5-6.4}{5.2}$
	$= 0.109$	A1		Correct prob
	P(1) $= (0.1091)^1(0.8909)^3 \times 4C1$	M1		Binomial term ${}_4C_x p^x(1-p)^{4-x}$ any p $x \neq 0$
	$= 0.309$ or 0.308	A1	[4]	Correct answer

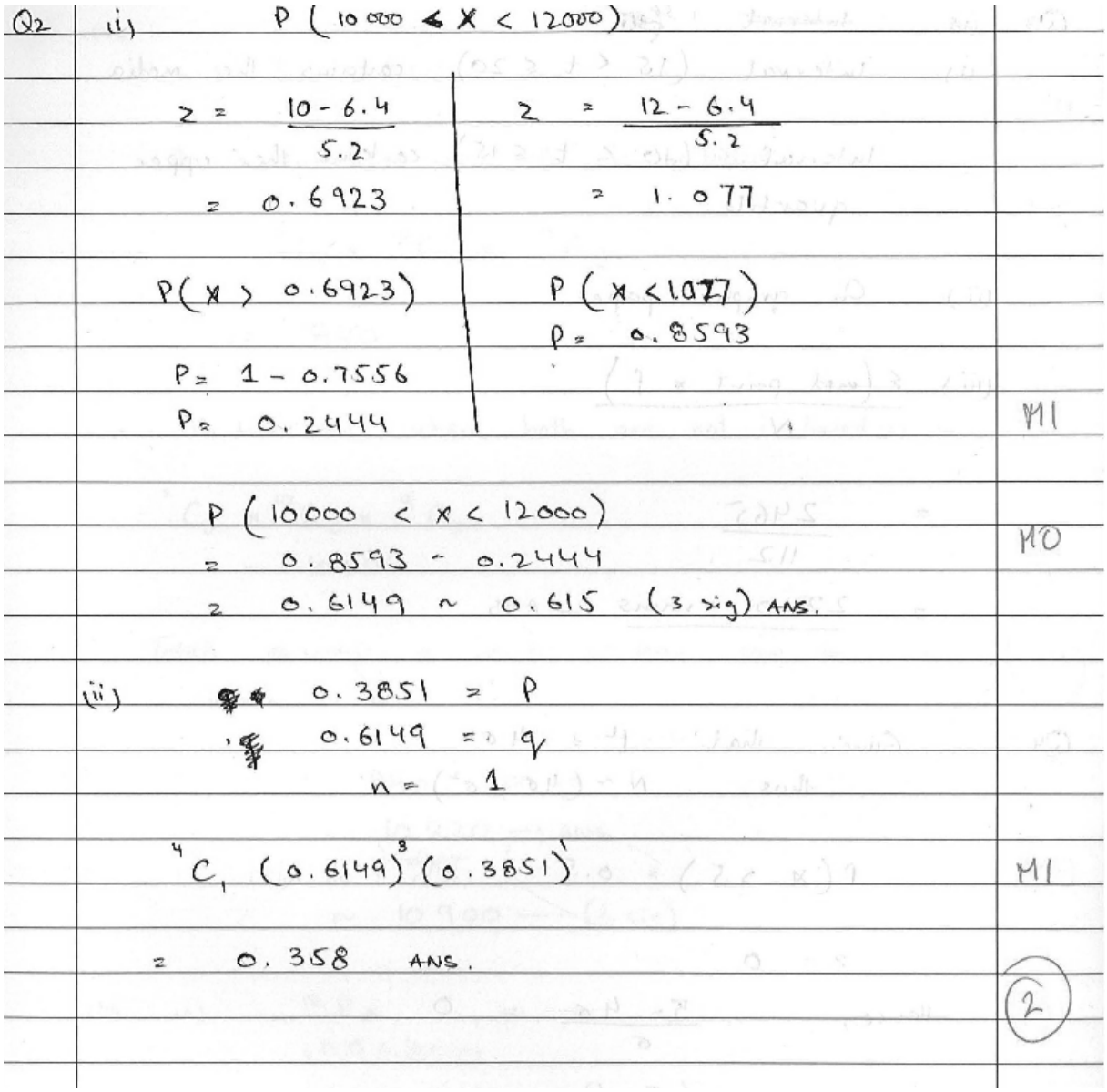
Example candidate response – 1



Item marks awarded: (i) = 3/3; (ii) = 1/4

Total mark awarded = 4 out of 7

Example candidate response – 2



Item marks awarded: (i) = 1/3; (ii) = 1/4

Total mark awarded = 2 out of 7

Examiner comment – 1 and 2

(i) The units, measured in thousands of dollars, posed problems to some candidates who did not realise that $\$10{,}000$ actually meant 10 when standardising. Candidate 1 mixed units originally, but realised that the value of z thus obtained was not sensible, so crossed the working out and used the correct values, gaining full marks for this part of the question. Candidate 2 managed to standardise correctly but was unable to find the correct area of the normal curve. Using a diagram would have helped to determine whether the required probability was sensible.

(ii) Both candidates correctly recognised the binomial situation but were unable to find the probability of making a loss. They did not appreciate that making a loss is the same as making a profit of 0 or less. Both candidates thought they should use their previous answer in some way, which they did and thus gained a method mark for the binomial attempt.

Question 3

3 The table summarises the times that 112 people took to travel to work on a particular day.

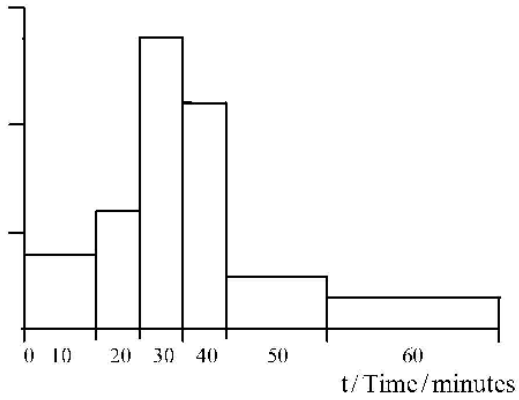
Time to travel to work (t minutes)	$0 < t \leqslant 10$	$10 < t \leqslant 15$	$15 < t \leqslant 20$	$20 < t \leqslant 25$	$25 < t \leqslant 40$	$40 < t \leqslant 60$
Frequency	19	12	28	22	18	13

(i) State which time interval in the table contains the median and which time interval contains the upper quartile. [2]

(ii) On graph paper, draw a histogram to represent the data. [4]

(iii) Calculate an estimate of the mean time to travel to work. [2]

Mark scheme

3 (i) median in 15–20 mins, UQ in 25–40 mins	B1 B1	[2]	
(ii) fd 1.9, 2.4, 5.6, 4.4, 1.2, 0.65 or Scaled freq 9.5, 12, 28, 22, 6, 3.25  	M1 A1 B1 B1	[4]	Attempt at fd or scaled freq [f / (attempt at cw)] Correct heights seen on diagram Correct bar widths visually no gaps Labels (time / mins and fd or freq per 5 min) and correct bar ends
(iii) $(5 \times 19 + 12.5 \times 12 + 17.5 \times 28 + 22.5 \times 22 + 32.5 \times 18 + 50 \times 13)/112 = 2465/112$ $= 22.0$ minutes	M1 A1	[2]	Attempt at $\Sigma xf / 112$ using mid-points, NOT classwidths, NOT upper class bounds Correct answer accept 22

Example candidate response – 1

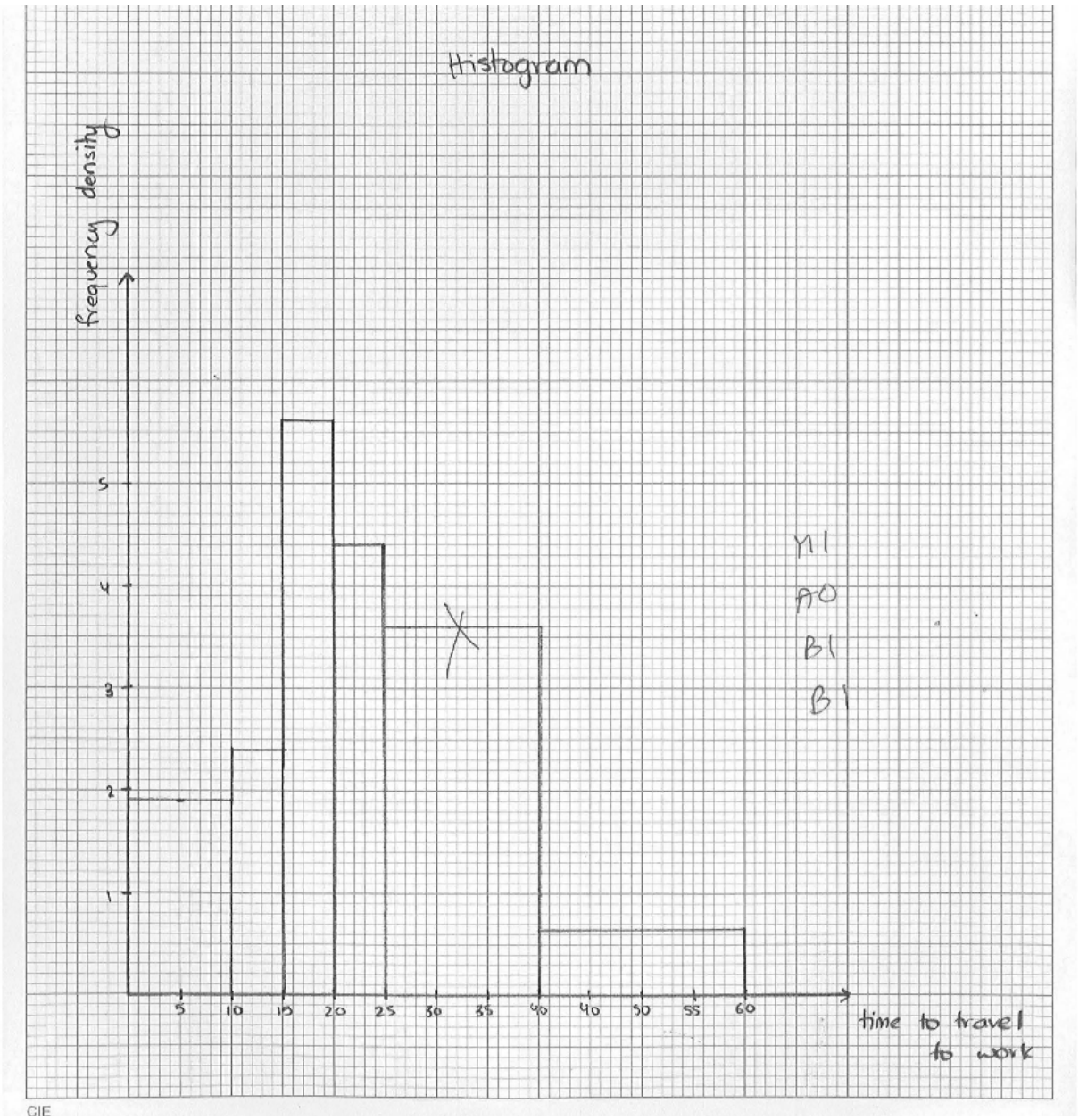
3 i median → $15 < t \leqslant 20$

Upper quartile → ~~$20 <$~~ $20 < t \leqslant 25$ B1

3 ii frequency densities

$\frac{19}{10}$, $\frac{12}{5}$, $\frac{28}{5}$, $\frac{22}{5}$, $\frac{18}{15}$, $\frac{13}{20}$

1.9 2.4 5.6 4.4 3.6 0.65



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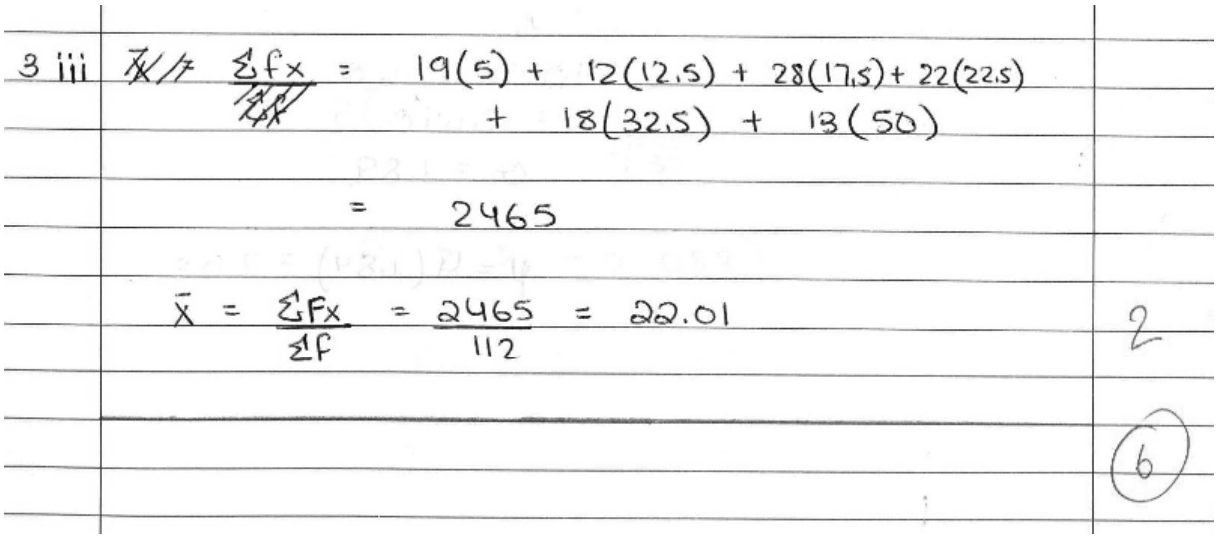
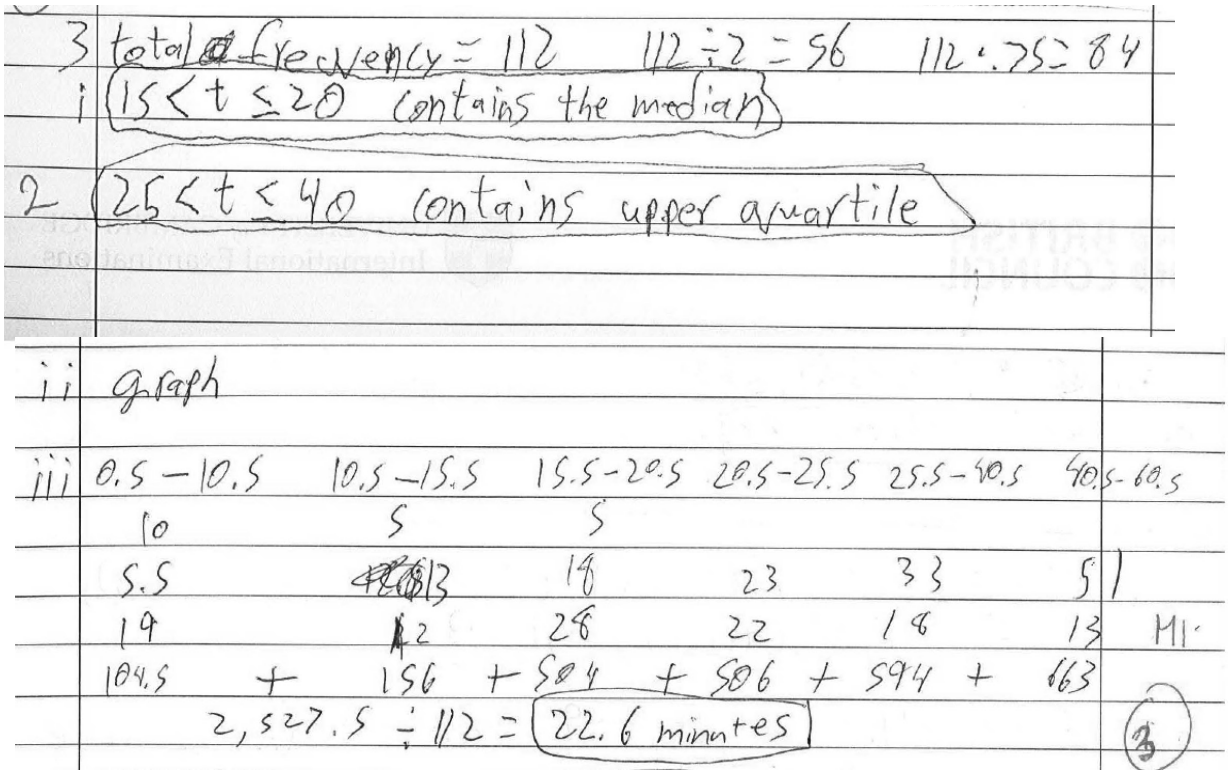
Example candidate response – 1, continued



Item marks awarded: (i) = 1/2; (ii) = 3/4; (iii) = 2/2

Total mark awarded = 6 out of 8

Example candidate response – 2



3 i total frequency = 112 $112 \div 2 = 56$ $112 \times .75 = 84$

$15 < t \leq 20$ contains the median

2 $25 < t \leq 40$ contains upper quartile

ii graph

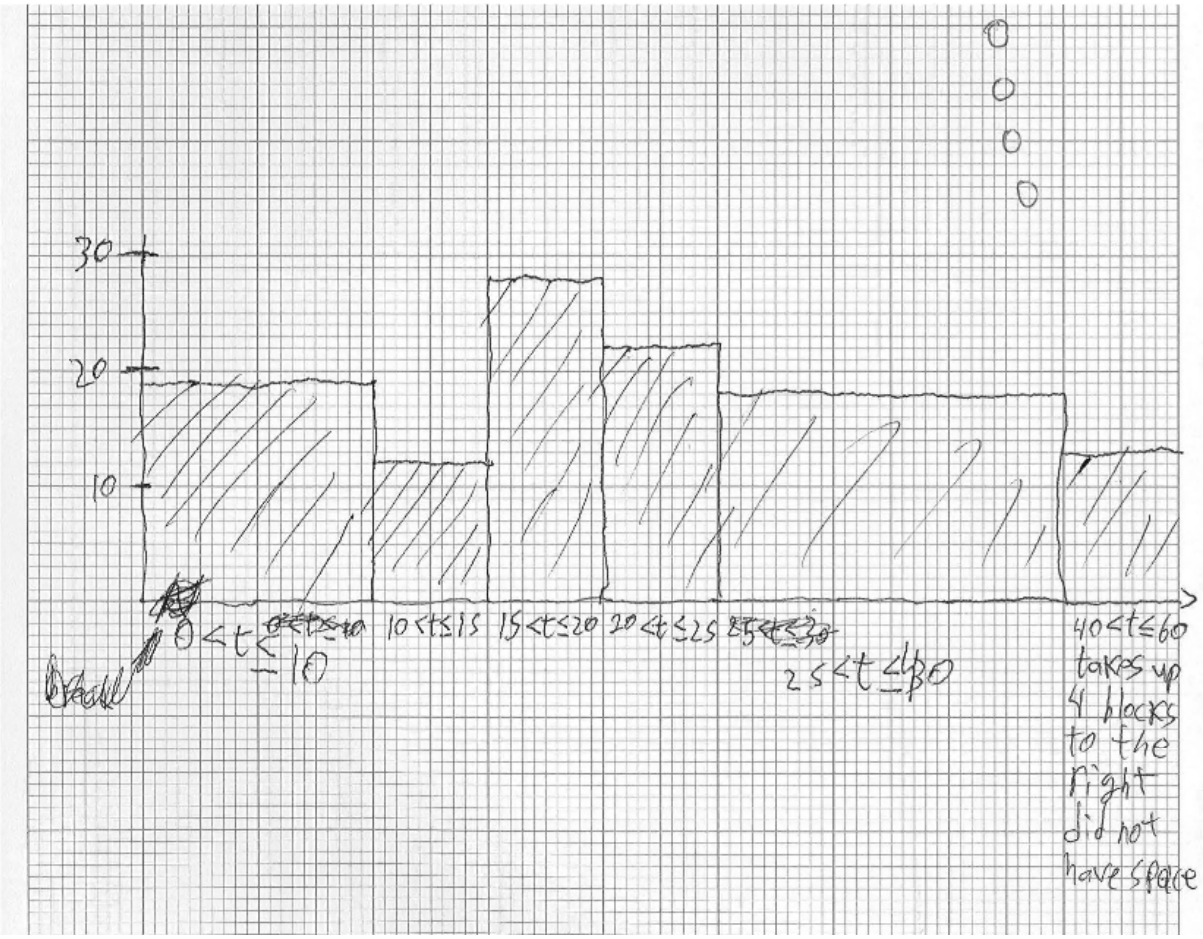
iii

0.5 – 10.5	10.5 – 15.5	15.5 – 20.5	20.5 – 25.5	25.5 – 40.5	40.5 – 60.5	
10	5	5				
5.5	13	18	23	33	51	
19	12	28	22	16	13	M1
104.5	+ 156	+ 504	+ 506	+ 594	+ 663	

$2,527.5 \div 112 =$ 22.6 minutes

3

Example candidate response – 2, continued



Item marks awarded: (i) = 2/2; (ii) = 0/4; (iii) = 1/2

Total mark awarded = 3 out of 8

Examiner comment – 1 and 2

(i) Candidate 1 showed no working for part **(i)** and made a mistake, whereas candidate 2 got this part completely correct.

(ii) The graph was well done by candidate 1 who found the frequency densities correctly, labelled the axes correctly but plotted one of the heights on the graph at 3.6 instead of 1.2. Candidate 2 plotted frequencies instead of frequency densities, a very common mistake. Even so, the candidate could have gained a mark if the widths of the bars had all been correct visually. This candidate did not label the axes correctly, chose an inappropriate scale and was therefore unable to fit the entire graph on the page.

(iii) Candidate 1 found the mean correctly choosing the correct mid-points of the intervals. Candidate 2 thought the intervals went from 0.5 to 10.5 and so on, instead of 0 to 10, but was otherwise mainly correct and so was awarded a method mark but no accuracy mark.

Question 4

4 The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

(i) Find the mean and standard deviation. [4]

(ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5. [5]

Mark scheme

4	**(i)** $z = 1.036$ or 1.037	B1		± 1.036 or ± 1.037 seen
	$1.036 = \frac{5-4s}{s}$	B1		$\frac{5-4\sigma}{\sigma}$ seen or $\frac{5-\mu}{\mu/4}$ oe
	$s = 0.993$	M1		One variable and sensible solving attempt z-value not nec
	$\mu = 3.97$	A1		Both answers correct
			[4]	
	(ii) $p = 0.85$			
	$\mu = 200 \times 0.85 = 170$,	B1		200×0.85 (170) and $200 \times 0.85 \times 0.15$ (25.5) seen
	$\text{var} = 200 \times 0.85 \times 0.15 = 25.5$	M1		Standardising, sq rt and must have used 200
	$P(\text{at least } 160) = P\left(z > \frac{159.5-170}{\sqrt{25.5}}\right)$	M1		continuity correction 159.5 or 160.5
	$= P(z > -2.079)$	M1		correct area (> 0.5) must have used 200
	$= 0.981$	A1		correct value
			[5]	

Example candidate response – 1

Q4. $\mu = 4(\sigma)$ $\sigma = ?$

$P(X > 5) = 0.15$

i) $\mu = ?$

$P\left(\frac{X-\mu}{\sigma} < \frac{5-\mu}{\sigma}\right) = 0.15$ (using the notation of z)

$P\left(z < \frac{5-4\sigma}{\sigma}\right) = 0.15.$

or

$P(z < /5\sigma$

AS $z >$ hence,

B0 $1 - \Phi\left(\frac{5-4\sigma}{\sigma}\right) = 0.15.$

$1 - 0.15 = \Phi\left(\frac{5-4\sigma}{\sigma}\right).$

B1 $0.85 = \Phi\left(\frac{5-4\sigma}{\sigma}\right)$

$1 - \left(1 - \Phi\frac{5-4\sigma}{\sigma}\right) = 0.15$

$1 - 1 + \Phi\left(\frac{5-4\sigma}{\sigma}\right) = 0.15$

M1

1.3006 $1.36 = \frac{5-4\sigma}{\sigma}$

$1.36\sigma = 5 - 4\sigma$

$5.36\sigma = 5.$

$\sigma = 5/5.36 = 0.933$ (3 s.f)

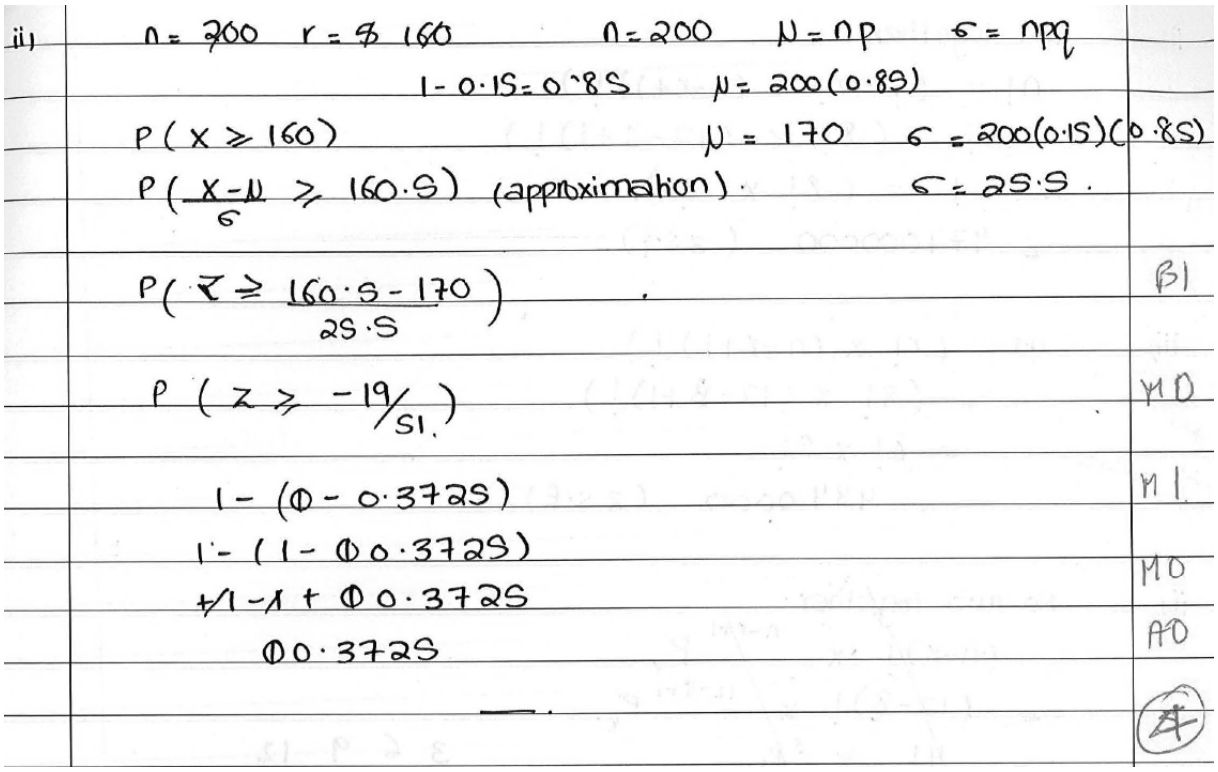
Now,

$\mu = 4(\sigma)$

$\mu = 4(0.933)$

$\mu = 3.73$ (3 s.f).

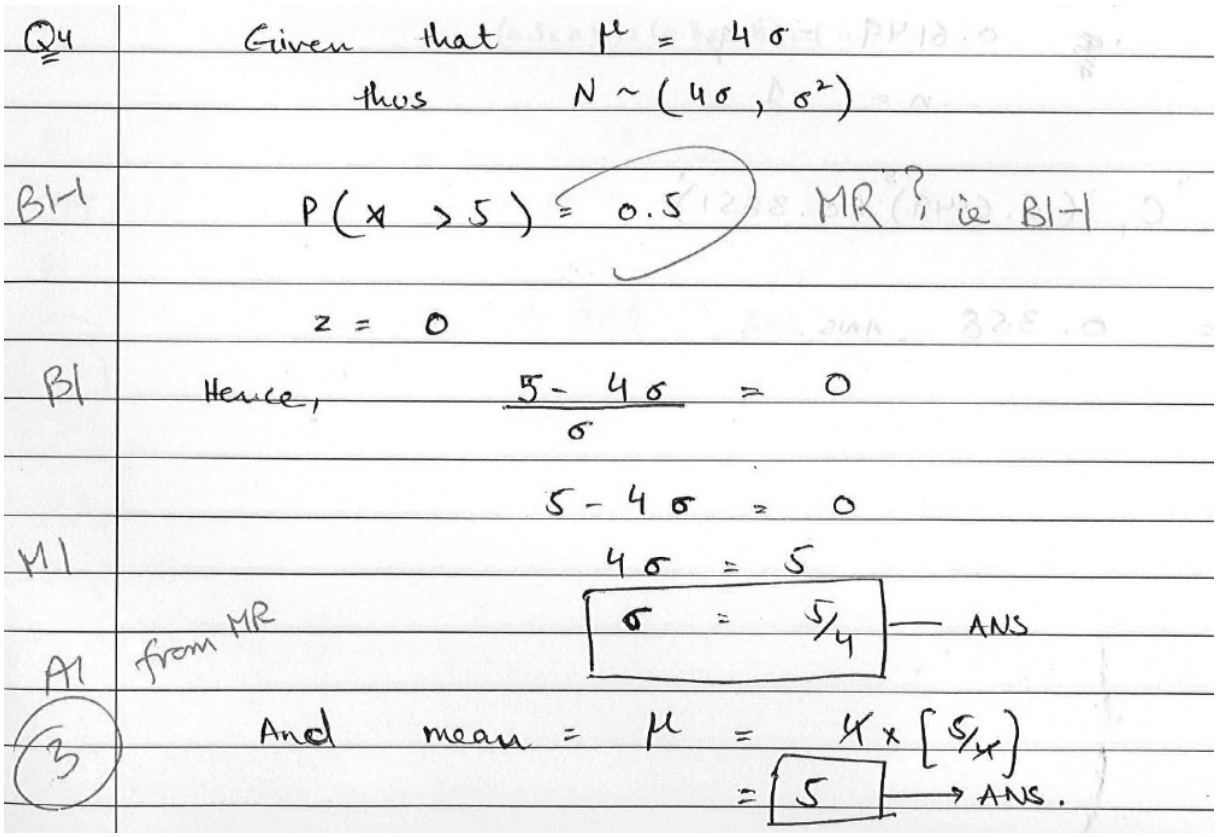
Example candidate response – 1, continued



Item marks awarded: (i) = 2/4; (ii) = 2/5

Total mark awarded = 4 out of 9

Example candidate response – 2



Item marks awarded: (i) = 3/4; (ii) = 0/5

Total mark awarded = 3 out of 9

Examiner comment – 1 and 2

(i) Candidate 1 used the normal tables backwards to find $\Phi^{-1}(0.85)$ but wrote 1.36 instead of 1.036. Candidate 2 obtained the correct z-value. Both candidates sorted out the information correctly regarding the mean being four times the standard deviation, and gained a method mark for attempting to solve their resulting equation. Candidate 2 would have gained full marks for part **(i)** if their answer had been written correct to three significant figures instead of only two.

(ii) Candidate 1 recognised the normal approximation to the binomial and selected the correct probability, 0.85, of being less than 5. Candidate 1 did not use a square root when standardising, although did use a continuity correction but chose the wrong area of the normal curve, thus not being awarded two method marks. Candidate 2 did not recognise the normal approximation to the binomial and tried to use the binomial probabilities to find $P(X = 160)$ but could not find $P(X > 160)$. Thus no marks could be gained.

Question 5

5 **(a)** A team of 3 boys and 3 girls is to be chosen from a group of 12 boys and 9 girls to enter a competition. Tom and Henry are two of the boys in the group. Find the number of ways in which the team can be chosen if Tom and Henry are either both in the team or both not in the team. [3]

(b) The back row of a cinema has 12 seats, all of which are empty. A group of 8 people, including Mary and Frances, sit in this row. Find the number of different ways they can sit in these 12 seats if

(i) there are no restrictions, [1]

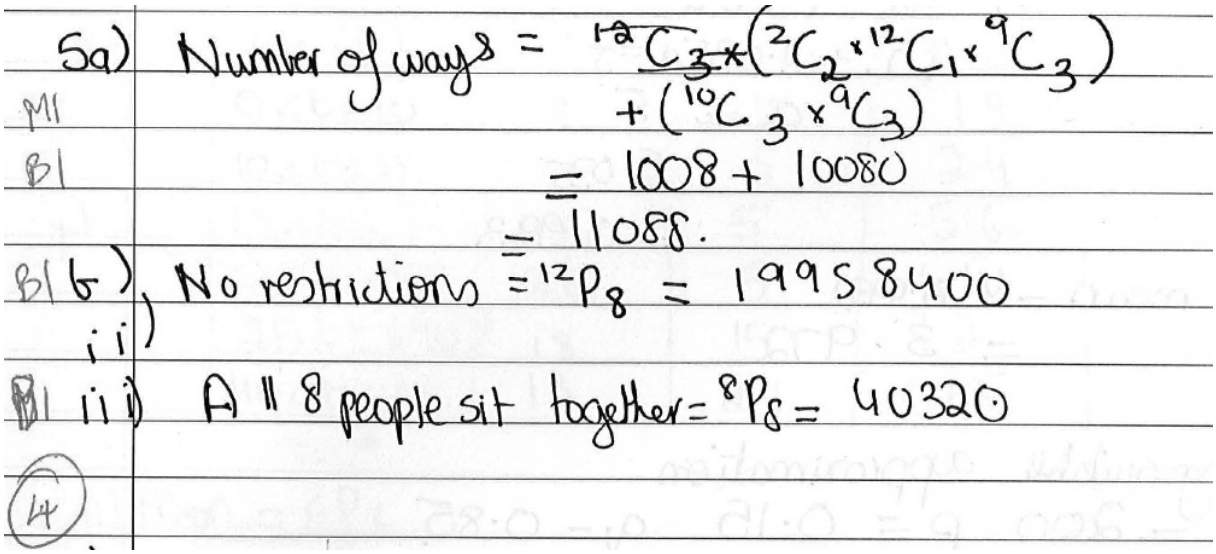
(ii) Mary and Frances do not sit in seats which are next to each other, [3]

(iii) all 8 people sit together with no empty seats between them. [3]

Mark scheme

5(a)	Boys in:10C1 × 9C3 = 840 ways Boys out: 10C3 × 9C3 = 10080 ways Total = 10920 ways (10900)	M1 B1 A1	[3]	summing two 2-factor products, C or P Any correct option unsimplified Correct final answer
(b)(i)	$_{12}P_8$ = 19,958,400	B1	[1]	or 20,000,000
(ii)	together: $_{11}P_7$ = 1663200 × 2 = 3326400 Not tog: 19958400 – 3326400 =16,632,000 (16,600,000) OR M at end then not F in 10 × 10P6 × 2=3024000 ways not at end in 10 × 9 × 10P6 = 13608000 ways Total = 16,632,000 ways	B1 M1 A1 M1 B1 A1	[3]	$_{11}P_7$ seen 19958400 or their (i) – their together (must be >0) correct final answer summing options for M at end and M not at end one correct option correct final answer
(iii)	8! × 5 = 201600 ways	B1 M1 A1	[3]	8! seen mult by equivalent of integer ≥ 1 Mult by 5 Correct answer SR 8! × 5!=4838400 B2

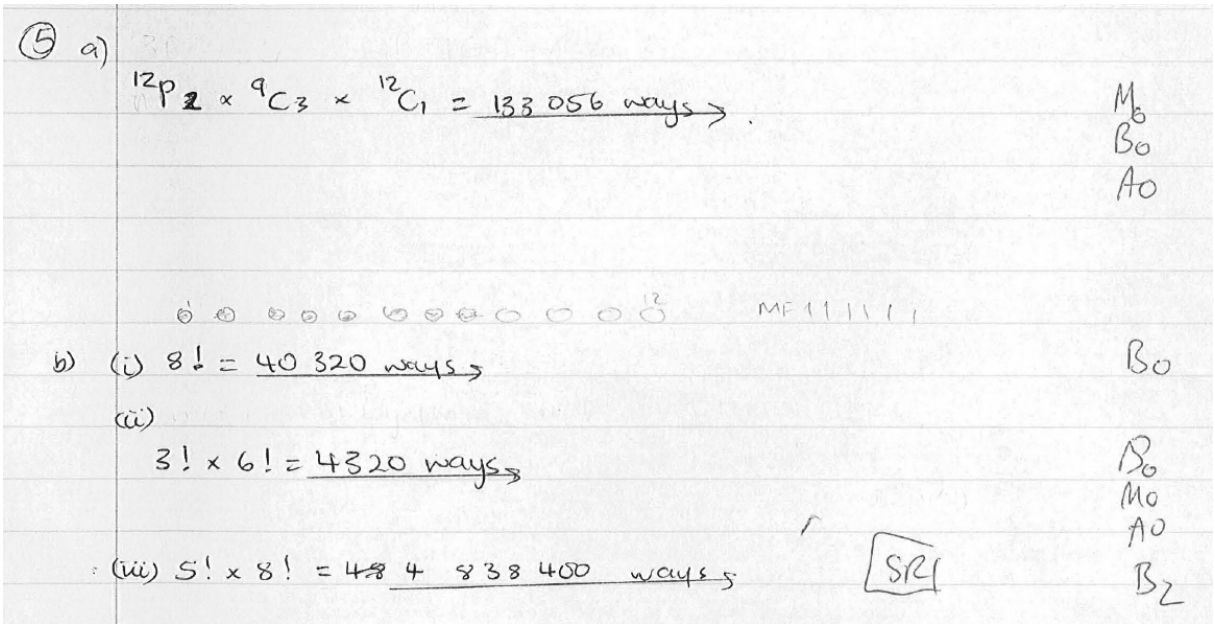
Example candidate response – 1



Item marks awarded: (a) = 2/3; (b)(i) = 1/1; (ii) = 0/3; (iii) = 1/3

Total mark awarded = 4 out of 10

Example candidate response – 2



Item marks awarded: (a) = 0/3; (b)(i) = 0/1; (ii) = 0/3; (iii) = 2/3

Total mark awarded = 2 out of 10

Examiner comment – 1 and 2

(a) Candidate 1 appreciated that two options had to be added, namely boys in and boys out. One of the options was correct, and a method mark was awarded for adding the two options. Candidate 2 knew about permutations and combinations, but was unable to apply their knowledge correctly.

(b) **(i)** This 1 mark question was answered correctly by candidate 1 but not by candidate 2.

(ii) Neither candidate could make any headway in this part of the question.

(iii) Both candidates appreciated that the 8 people could be arranged in 8! different ways. One forgot about the spaces and one thought the spaces could be arranged in 5! different ways.

Question 6

6 A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable X is the sum of the three scores.

(i) Show that $P(X = 9) = \frac{10}{64}$. [3]

(ii) Copy and complete the probability distribution table for X. [3]

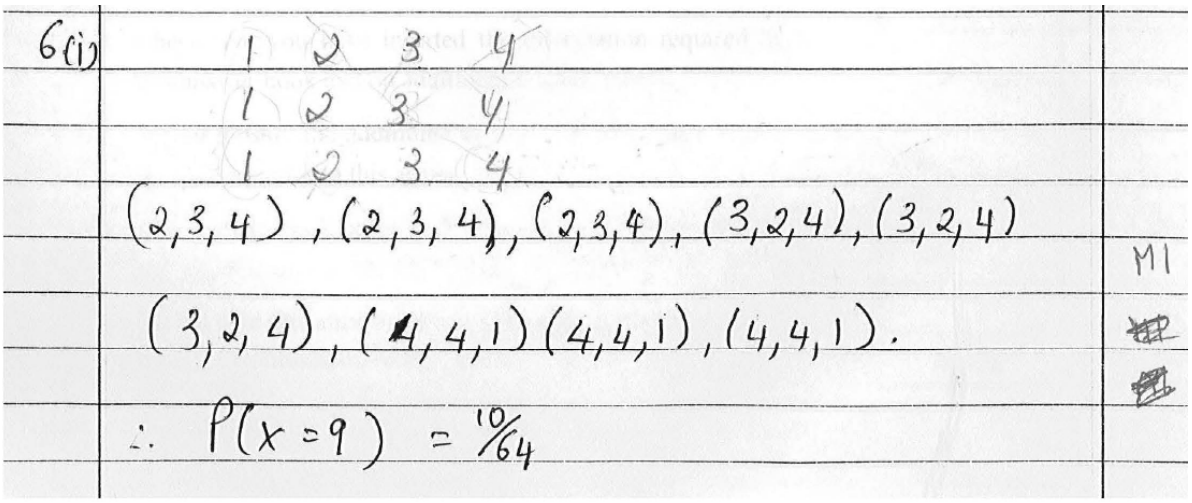
x	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{64}$	$\frac{3}{64}$			$\frac{12}{64}$					

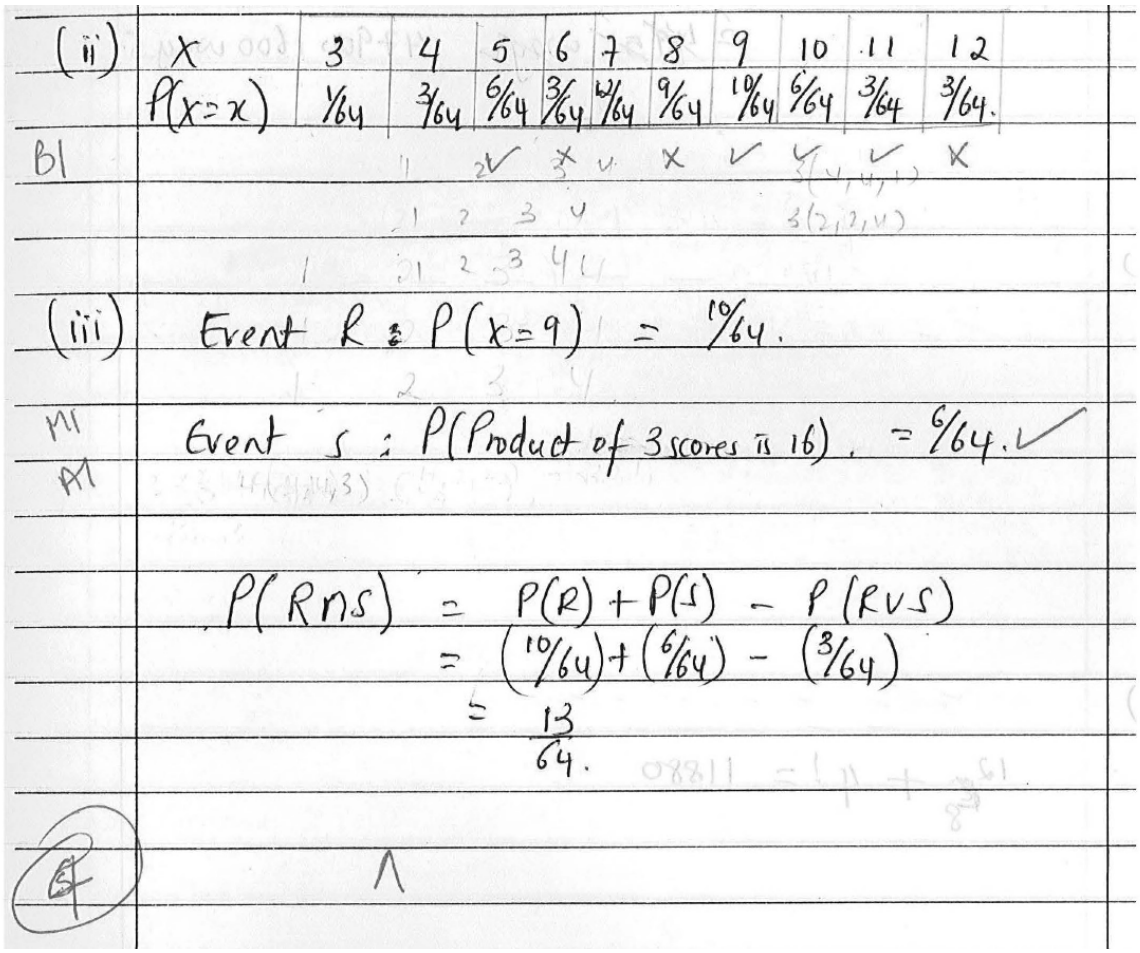
(iii) Event R is 'the sum of the three scores is 9'. Event S is 'the product of the three scores is 16'. Determine whether events R and S are independent, showing your working. [5]

Mark scheme

6	**(i)** $P(9) = P(1,4,4) \times 3 + P(2,3,4) \times 6 + P(3,3,3)$	M1 M1		Listing at least 2 different options Multiplying P(4,3,2) by 6 or P(1,4,4) by 3
	$= 10/64$ (5/32) (0.156) AG	A1	[3]	Correct answer must see numerical justification
	(ii) probs 1/64, 3/64, 6/64, 10/64, 12/64, 12/64, 10/64, 6/64, 3/64, 1/64.	B1 B1 B1	[3]	3 or more additional correct probs 5 or more correct All correct
	(iii) $P(S) = 6/64(3/32)$	M1		An attempt at $P(S)$ 4,4,1 or 4,2,2
	$P(R \cap S) = 3/64, \neq 15/1024$ ie $P(R) \times P(S)$	A1 B1		Correct $P(S)$ Correct $P(R \cap S)$ in either intersection or cond prob cases
	OR $P(R\|S) = \frac{3/64}{6/64} = 1/2, \neq 10/64$ ie $P(R)$	M1		comparing their $P(R \cap S)$ with their $P(R) \times P(S)$ or their $P(R\|S)$ with their $P(R)$ need numerical vals
	Not independent	A1ft	[5]	correct conclusion ft wrong $P(S)$ or $P(R \cap S)$ only

Example candidate response – 1

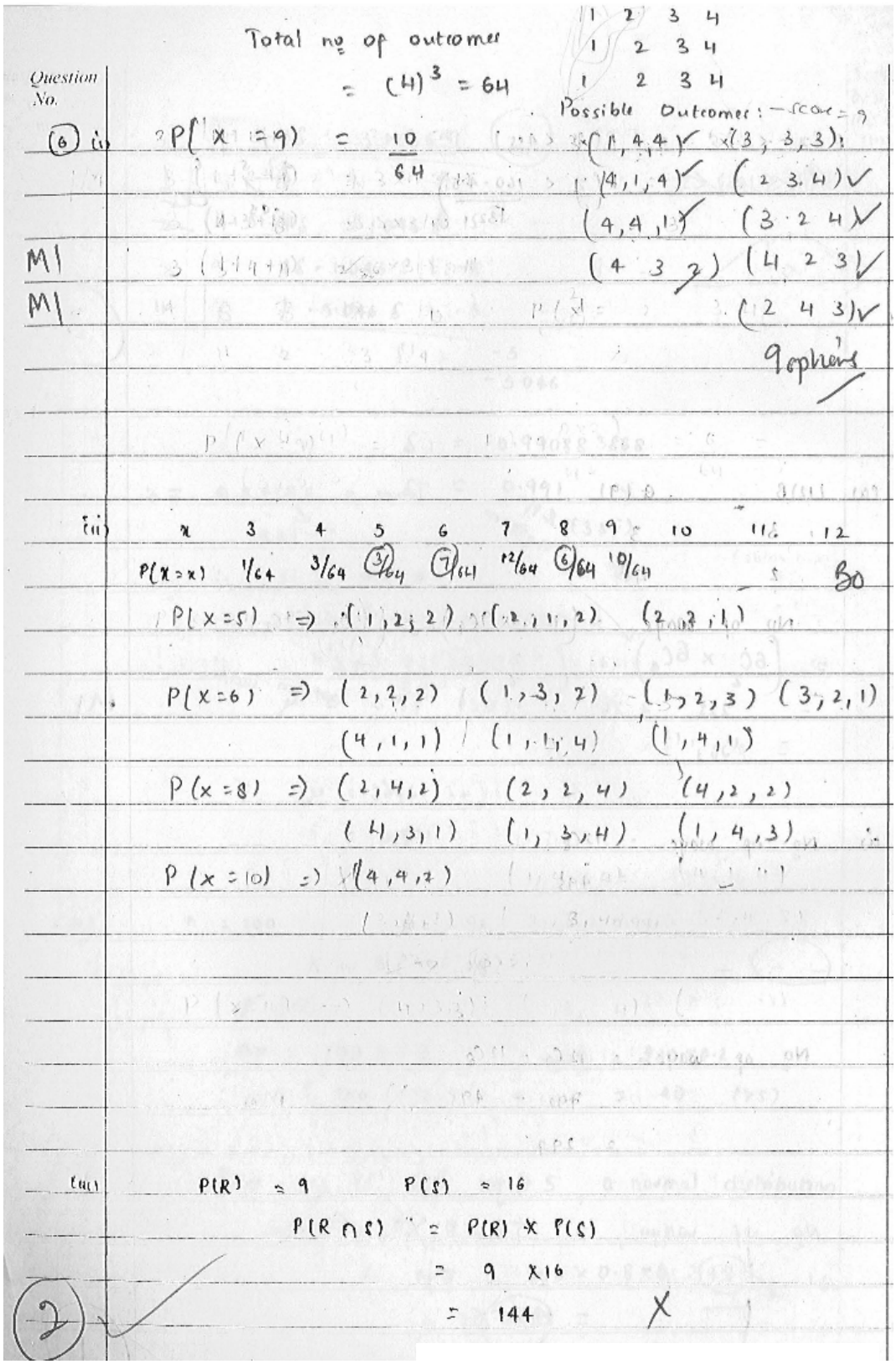




Item marks awarded: (i) = 1/3; (ii) = 1/3; (iii) = 2/5;

Total mark awarded = 4 out of 11

Example candidate response – 2



Item marks awarded: (i) = 2/3; (ii) = 0/3; (iii) = 0/5;

Total mark awarded = 2 out of 11

Examiner comment – 1 and 2

(i) Both candidates tried to find 10 options. Candidate 1 found (2, 3, 4) and (4, 4, 1) but did not realise that (2, 3, 4) was different from (3, 2, 4) and so on. Candidate 2 found 9 options including (3, 3, 3) but omitted the tenth (3, 2, 4).

(ii) The probability distribution table was copied from the question paper. Candidate 1 had four correct solutions, whereas candidate 2 missed many of the options and only had one correct solution.

(iii) Candidate 1 realised that they had to find P (product of the 3 scores is 16) and found it correctly but could not remember the definition of independence. Candidate 2 could not make any headway in this last part of the question.

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